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Study of the Vertical Scattering Properties of the Lithosphere Based on the Inversion of *P*– and *S*–Wave Pulse Broadening Data

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The vertical distribution of the scattering properties of the lithosphere can be found using Bocharov's formula which describes the broadening of an incoherent pulse with distance as it propagates in a scattering medium with scattering varying along the raypath. For a piecewise constant scattering distribution, Bocharov's formula yields a linear equation for every raypath that relates the scattering of the blocks traversed by the raypath to a parameter that defines the pulse width. This set of equations can be solved by least squares to find scattering distribution in a certain lithospheric volume. Records of *P* and *S* waves from local earthquakes with focal depths of up to 140 km made at six Kamchatkan stations were measured to determine the onset-to-peak delay times (400 – 600 measurements). These data were inverted for each station to find scattering for a layer-on – half-space model. The results did not differ much between the stations and yielded the following distribution: scattering was 0.02 km^{-1} (*P* waves) and 0.008 km^{-1} (*S* waves) in the upper layer 30–50 km thick and 5 – 10 times as low in the lower layer ((30...50) – 140 km). The exact scattering of the lower layer was not evaluated.

INTRODUCTION

The vertical distribution of the earth scattering properties is virtually unknown beyond the general idea of scattering rapidly decreasing with depth. Variation observed in teleseismic *P*–wave phases and amplitudes were used to estimate the thickness of the near–surface scattering layer 100–250 km thick and to determine the types of parameters that characterize the scattering properties of the crust and upper mantle [8], [9], [10]. However, the scattering values thus obtained were an order of magnitude lower than those derived from *S* waves generated by local earthquakes [11], [12], [13]. Because these records were interpreted using scattering models for a statistically homogeneous earth, this difference might be caused by a number of factors.

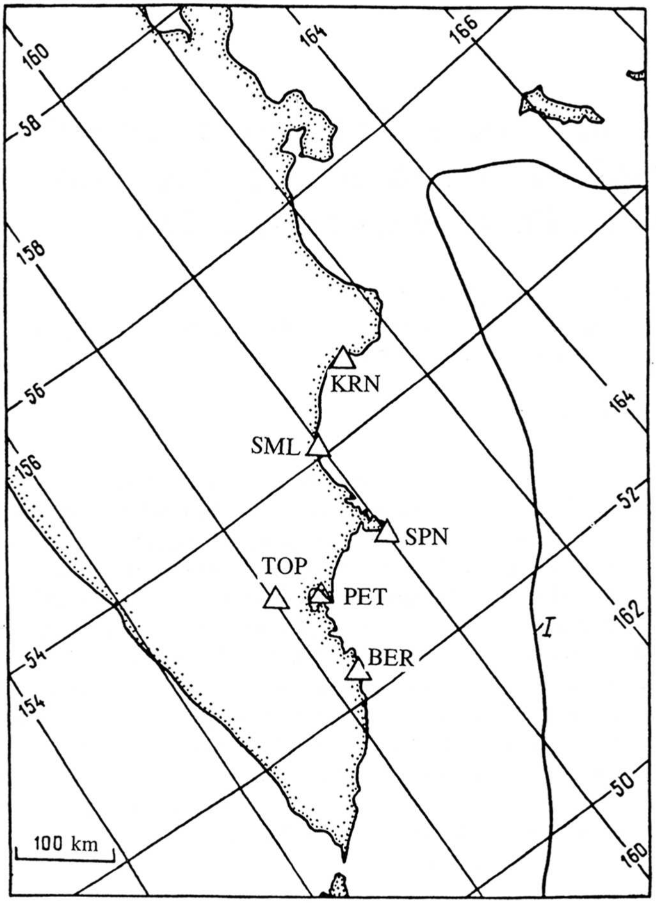

Figure 1 Location of six regional seismic stations in Kamchatka, for which vertical scattering distribution was studied. Station codes and names: BER – Berezovaya, TOP – Topolovo, PET – Petropavlovsk, SPN – Shipunskiy, SML – Semlyachik, KRN – Kronoki. I – trench axis.

This study was an attempt to estimate the vertical distribution of scattering (scattering coefficient) in the lithosphere of Kamchatka, proceeding from the known fact that a power pulse radiated by an instantaneous source broadens as it propagates in a scattering medium [5]. Gusev and Lemzikov [5] used a simple data processing sequence to obtain first estimates for the average scattering g of S-waves which were in satisfactory agreement with the estimates derived from the amplitude ratios of direct and scattered wave signals.

Earlier [1] we used the Monte Carlo technique to simulate the pulse broadening process for forward scattering and used the results to interpret observed t_m data, i.e., the delay of the apparent peak in an S wave group relative to the onset of that group (i.e., the

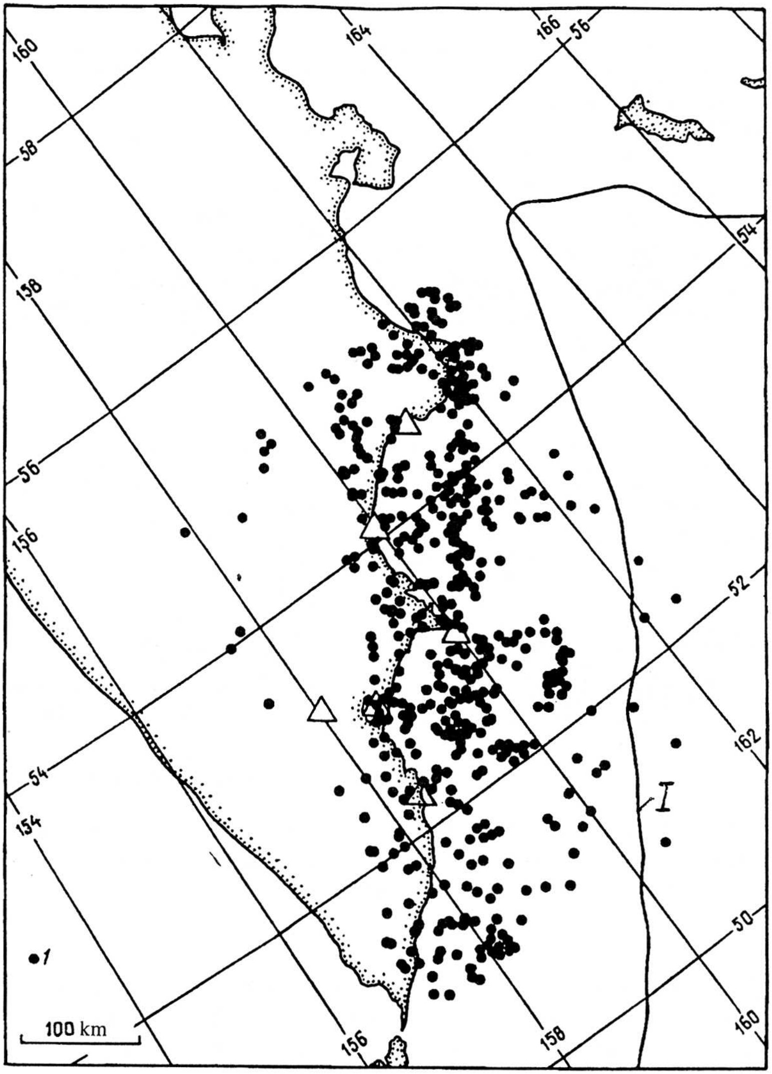

Figure 2 The epicenters of earthquakes (*1*) used in this study to derive vertical scattering distributions. For the other notation see Fig. 1.

traveltime for the shortest path). We obtained more accurate average scattering estimates: the values for sources in the depth ranges 0–40 and 40–90 km differed by a factor of 2. This great difference encouraged us to investigate a vertical scattering distribution.

The theoretical principles of the approach we used were laid down in [2], [3], [7], [15]. Williamson [15] derived a formula describing the shape of a power pulse radiated

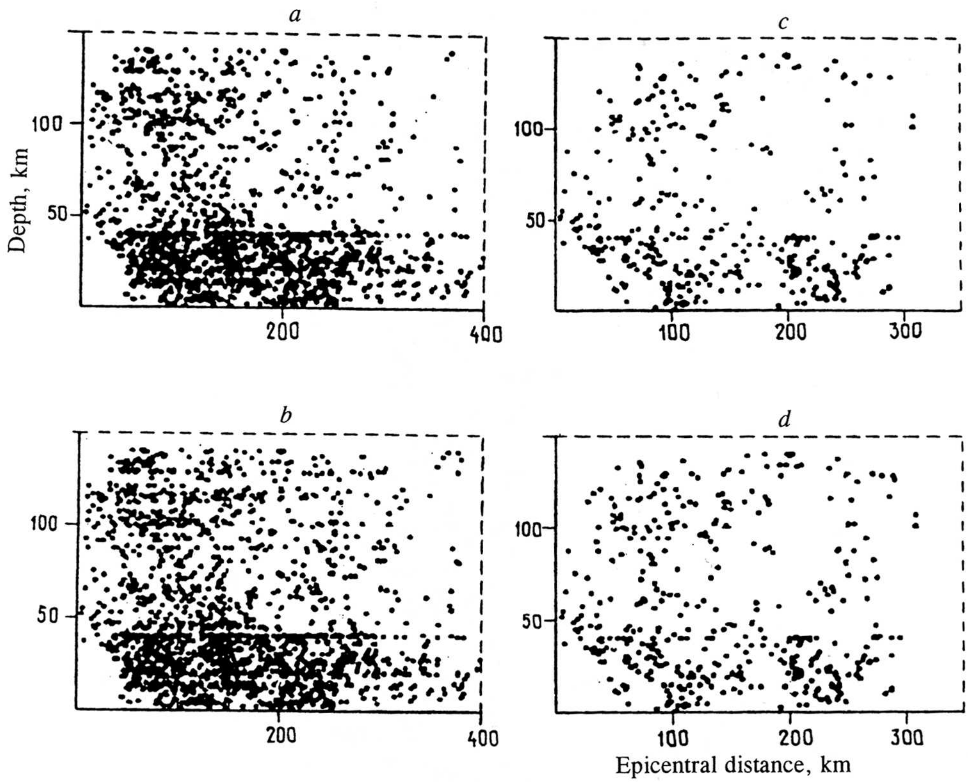

Figure 3 Hypocenters of earthquakes used to derive the vertical scattering distribution shown in the depth – epicentral distance plane: *a, b* – data from all stations for *P* and *S* waves, respectively; *c, d* – data from SPN for *P* and *S* waves, respectively.

by an instantaneous source in a statistically homogeneous random medium that causes multiple small-angle scattering. The power peak delay predicted by this formula for short optical wavelengths are, as a matter of fact, in agreement with the results of our numerical modeling [7] up to wavelengths of about 2. Bocharov [2] derived a formula for estimating t_c (the delay of the center of gravity of a radiated pulse due to the propagation through a medium of varying scattering) and used it to work out an inversion method for determining a 3D scattering field, or to be more specific, the diffusion coefficient which has the meaning of the variance of angular scattering for a wave per unit raypath length [3]. The Bocharov formula leads to linear equations in unknown scattering and can be used to construct a practical algorithm for the inversion of seismological t_m observations when some extra assumptions are made. The main assumption we made was that the proportionality between t_c and t_m^0 (the average delay of the power pulse peak in a medium of uniform scattering) was approximately valid for nonuniform scattering as well. In other words, the power pulse shape was assumed to be almost independent of spatial scattering variation: the pulse merely stretches out as it travels, just as in the case of uniform scattering.

Lithospheric scattering of P and S waves in the frequency range 2-6 Hz can be estimated from t_m values measured on records of small local earthquakes, when treated as experimental t_m^0 estimates. Although these data involve large random errors, when the sample is large enough, one can estimate the trend of t_m variation with increasing distance r, average scattering [7], or other scattering parameters [14], and even, as has been mentioned, reveal differences between the $t_m(r)$ trends for earthquakes of different focal depths. The above arguments constitute the principles of the approach we used.

THEORY

Assuming a small local earthquake to be an instantaneous point source of body waves in the frequency range 2–6 Hz, we will follow Bocharov [2] who showed that the average delay of a pulse undergoing multiple scattering at large heterogeneities in a random medium (a medium that is statistically nonuniform along the line of sight connecting the source and receiver) can be written in the form

$$t_c(r) = \frac{1}{2c}\int_0^r (r - r_1)\frac{r_1}{r} D(r_1)\,dr_1, \tag{1}$$

where c is the velocity of wave propagation in the medium, and D is the diffusion coefficient varying in space:

$$D(r) = \beta(r)\,\theta_0^2(r), \tag{2}$$

where β (r) is the scattering coefficient, and θ_0^2 (r) the average squared scattering angle at a heterogeneity. The coefficient β (r) is related to the scattering g_a (r) for anisotropic scattering as

$$g_{\mathrm{a}}(r) = 1/\beta(r). \tag{3}$$

In our previous papers [5], [7] we mentioned that it was difficult to estimate g_{a} (r) and θ_0^2 (r) separately, while the diffusion coefficient D can be related, because θ_0^2 (r) is small where heterogeneities are large, to equivalent isotropic scattering g_e:

$$D(r) = \theta_0^2(r)/g_{\mathrm{a}}(r) \approx 2\,[1 - \langle\cos(\theta_0(r))\rangle\,]/g_{\mathrm{a}}(r) \equiv 2g_e(r), \tag{4}$$

where $\langle\cos(\theta_0(r))\rangle$ is the mean cosine of the scattering angle at the heterogeneity in question. The parameter $g_e(r) = (1 - \langle\cos(\theta_0(r))\rangle\,)/g_{\mathrm{a}}(r)$ is related to diffusional asymptotics scattering (see, e.g., [6]); it can be and was often found from the relative amplitudes of direct and coda waves. This parameter is also related to pulse broadening. In our previous paper [7] we demonstrated by straightforward numerical modeling that $t_m^0(r) \approx 0.1g_e r^2/c$ in a statistically uniform scattering medium, even if the condition $\theta_0^2 \ll 1$ does not hold.

The above considerations suggest that a meaningful 3D g_e structure can be obtained by inverting t_m data, even when the assumption of a large heterogeneity does not hold in a real medium. Below g_e will be simply called scattering.

In a statistically uniform, random scattering medium, the shape of a pulse (when averaged over the medium) generated by an instantaneous point source is given by [15]

$$I(r,t) = 2\pi^2 c/(g_e r^2) \sum_{n=1}^{\infty} (-1)^{n-1} n^2 \exp[-n^2 (t - r/c)\pi^2 c/(g_e r^2)]. \tag{5}$$

This formula gives the following expressions for some pulse duration measures:
– the first moment (center of gravity)

$$t_c = g_e r^2/(6c); \tag{6}$$

- the square root of the second central moment (rms duration)

$$T_{\mathrm{c(rms)}} = g_e r^2/(3\sqrt{10}\,c); \tag{7}$$

- the delay of the peak relative to the onset

$$t_m^0 = g_e r^2/(10.9c). \tag{8}$$

Thus, in the case of a statistically uniform medium, t_c can be estimated from t_m^0 by the simple multiplication of the latter by 10.9/6. Following Bocharov [3], we assume that this can be applied as a first approximation, to a statistically nonuniform medium. Practical inversion will be carried out as follows. Rewrite (1) for the center of gravity of a pulse produced by the kth source in the form

$$(10.9/6)(t_m)_k / r_k = (r_k/c) \sum_{i \in I_k} \Phi_{ik} (g_e)_i = \sum_{i \in I_k} a_{ik} (g_e)_i + \varepsilon_k, \tag{9}$$

where r_k is the source–receiver distance; I_k is the set of all numbers of quasi–uniform blocks in the medium that contribute to $(t_m)_k$; $(g_e)_i$ is the effective scattering in the ith block; ε_k is a random error with a zero mean; and

$$\Phi_{ik} = \int_{\rho_{i-1}}^{\rho_i} (\rho - \rho^2)\, d\rho, \tag{10}$$

where $\rho_i = r_i/r_k$; r_i is the distance from the source to the point where the raypath emerges from the ith quasi-uniform block. The inversion is performed on the assumptions of a stratified medium and straight rayspaths.

Generally speaking, equation (9) could be multiplied or divided by an arbitrary power of r_k, which is equivalent to the introduction of weights. The specific form of (9) was chosen so that the weights of the data points would not depend on the distance. Equation (9) is written down for every measured value of $(t_m)_k$; the a_{ik} value is calculated using an assumed horizontally stratified medium with constant velocity c and the known values of focal depths and epicentral distances.

It should be noted that a simple correction for raypath curvature does not give a significant accuracy improvement. The largest distortion is expected where crustal sources are located at distances larger than 150 km; in that case the fastest ray grazes the Moho, while the bulk of the S energy propagates in the crustal waveguide.

Table 1 Estimates $g_{e,1}$ and $g_{e,2}$ of effective scattering g_e and their standard errors for model 1 – a half-space and model 2 – a 35-km layer on a half-space. The g_e values are in 10^{-4} km^{-1}.

Station	*Wave type*	*n*	$g_{e,1}^{hs}$	$g_{e,2}^{la}$	$g_{e,2}^{hs}$
SPN	*P*	615	175 ± 8	192 ± 8	-5 ± 34
BER	*P*	554	152 ± 7	160 ± 8	-23 ± 46
KRN	*P*	411	233 ± 11	244 ± 11	23 ± 68
PET	*P*	519	150 ± 7	157 ± 8	4 ± 47
TOP	*P*	549	178 ± 5	186 ± 6	17 ± 36
SML	*P*	287	230 ± 11	233 ± 12	55 ± 243
SPN	*S*	689	68 ± 4	80 ± 4	6 ± 12
BER	*S*	570	55 ± 2	57 ± 2	13 ± 13
KRN	*S*	412	93 ± 7	103 ± 8	27 ± 36
PET	*S*	580	61 ± 3	64 ± 4	18 ± 15
TOP	*S*	559	87 ± 3	88 ± 3	59 ± 18
SML	*S*	268	98 ± 6	105 ± 7	-275 ± 135

The unknown values of $(g_e)_i$ in layers assumed to have constant scattering are found by least squares: these are variance ε_k, rms deviations, and the correlation matrix of $(g_e)_i$ estimates.

SOURCE DATA AND RESULTS OF CALCULATION

The delays $(t_m)_k$ were measured on seismograms of small local earthquakes recorded by six stations of the Kamchatka regional network (Fig. 1). The instruments were galvanometrically recording, three-component seismographs. The paper-drive speed was 2 mm/s. The response function was flat in the frequency range 1–10 Hz, the apparent frequency f_a ranged between 1.5 and 6 Hz. As was shown earlier [5], [7], average scattering estimates do not depend much on the frequency in this range, so that wide-band records can be safely used. The Kronoki station had an abnormal f_a value (6–8 Hz), probably because the frequency band underwent resonant inflation by the upper crust beneath this station.

We measured $(t_m)_k$ on 1985–1988 records, separately for *P* and *S* waves and for each of the components. The identification of *P* and *S*-wave peaks and of *P* onsets was easy. The onsets of *S* waves were often fuzzy. For comparison we determined t_m for *S* waves from 150 records using arrivals visually observed on the records and those from the travel time curves. No systematic difference was found: differences between the estimates of average scattering based on these data sets ranged between 5 and 10%. We therefore used visually identified *S* onsets in most of this study.

A comparison of three t_m values from the same station revealed high random errors. For this reason as many as 400–600 t_m measurements for each station were used to get meaningful g_e estimates. Figure 2 shows the epicenters of the earthquakes used; Fig. 3 displays the distribution of the source data t_m (measurements) separately for *P* and *S* waves in the depth-epicentral distance plane, for all stations and for one of them, the SPN station taken as an example.

We determined g_e for a uniform half-space model as a preliminary step in the calculation process. The results are given in Table 1 and can be summarized as follows: (1) the average scattering variations between the stations ranged within 30%; (2) the average $g^{hs}_{e,1}/P$ scattering for P waves ranged between 0.015 and 0.023 km^{-1}; (3) the average $g^{hs}_{e,1}/S$ scattering for S waves was in the range of 0.0055–0.0098 km^{-1}, which agreed with our previous estimates [5], [7].

Table 2 Estimates $g_{e,3}$ of effective scattering g_e and their standard errors for model 3 – a layer of variable thickness on a half-space. The g_e values are in 10^{-4} km^{-1}.

Station	*Wave type*	$g^{la}_{e,3}$	*Layer thickness, km*	$g^{hs}_{e,3}$
SPN	*P*	197 ± 13	30	13 ± 30
BER	*P*	160 ± 8	35	-23 ± 46
KRN	*P*	332 ± 23	10	165 ± 17
PET	*P*	160 ± 8	30	23 ± 41
TOP	*P*	336 ± 10	10	127 ± 9
SPN	*S*	79 ± 4	40	-1 ± 14
BER	*S*	59 ± 26	25	22 ± 94
KRN	*S*	175 ± 15	10	39 ± 12
PET	*S*	124 ± 9	5	46 ± 4
TOP	*S*	103 ± 5	10	72 ± 5

At the second step calculations were made for a layer of fixed thickness lying on a half-space. The thickness of the layer was set equal to 35 km on the basis of some preliminary estimates (this value is commonly taken to be the average thickness of the crust). The main results are listed in Table 1. The standard deviations for the right–hand sides of (9) are still large owing to high noise level, but the large size of the data set enabled us to estimate the effective scattering in the layer $g^{la}_{e,2}$ with a relative error below 10%. The correlation coefficient between the scattering estimates in the layer $g^{la}_{e,2}$ and in the half-space $g^{hs}_{e,2}$ was 0.1–0.2. The dependence is thus weak.

The results can be summarized as follows.

1. The contrast between the $g^{la}_{e,2}$ and $g^{hs}_{e,2}$ values was large (more than 5 times). The uncertainty of $g^{hs}_{e,2}$ was great, $g^{hs}_{e,2}$ being zero to $>2\sigma$ in all cases but one. All occasional negative values of $g^{hs}_{e,2}$ were not different from zero significantly; the fact of their appearance can be treated as an indication of the large contrast mentioned above; they are meaningless from the physical point of view.

2. The correlation between the $g^{hs}_{e,1}$ values for P and S waves at different stations, noted in the half-space case, was obvious between the P- and S-based $g^{la}_{e,2}$ values for the layer. The highest $g^{la}_{e,2}$ values were observed for the KRN and SML stations, the lowest for BER and PET.

The presence of negative $g^{hs}_{e,2}$ values forced us to adjust the layer thickness. This was done at the third step, for model 3 – a layer of variable thickness lying on a half-space. The thickness was varied at 5-km intervals. The version that yielded the optimal residual variance is given in Table 2. One can see that the choice of a fixed layer thickness equal to 35 km is justified only as a first approximation. In many cases the thickness of ~ 10 km proved to be more acceptable for the single sharp discontinuity.

Table 3 Estimates $g_{e,4}$ of effective scattering g_e and their standard errors for model 4 – two layers of variable thickness on a half-space. The g_e values are in 10^{-4} km^{-1}.

Station	*Wave type*	$g_{e,4}^{\mathrm{la1}}$	*Thickness of layer 1, km*	$g_{e,4}^{\mathrm{la2}}$	*Thickness of layer 2, km*	$g_{e,4}^{\mathrm{hs}}$
SPN	*P*	206 ± 15	15	160 ± 23	35	6 ± 35
BER	*P*	152 ± 9	25	275 ± 68	35	-53 ± 49
KRN	*P*	296 ± 33	10	147 ± 26	50	13 ± 105
PET	*P*	189 ± 13	10	120 ± 14	50	-23 ± 70
TOP	*P*	234 ± 16	10	34 ± 16	40	46 ± 212
SPN	*S*	90 ± 8	10	69 ± 8	40	3 ± 13
BER	*S*	67 ± 4	10	46 ± 5	30	23 ± 12
KRN	*S*	169 ± 16	10	53 ± 16	30	0 ± 32
PET	*S*	119 ± 10	5	56 ± 9	15	32 ± 7
TOP	*S*	102 ± 5	10	74 ± 4	70	3 ± 64

These circumstances called for the examination of model 4 – two layers of variable thickness on a half-space, even though it was not quite clear whether the source data justified the choice of this detailed a 5-parameter model. The work with this model was the final, fourth step. Optimization in terms of the discontinuity depth was done by a straightforward search at intervals of 5 km in the depth range 5 to 40 km and at intervals of 10 km in the range 40 to 80 km (Table 3). The correlation coefficients for any two of the three scattering estimates were within ± 0.5. In all cases but one (BER, *P* waves) the scattering of the upper layer was found to be higher than that of the lower, in agreement with theoretical expectation. Of the five cases in which the discontinuity was at 5–10 km for the previous model, four cases yielded one more discontinuity at 40–70 km depth, below which the scattering was low. It was only for *S* waves at the PET that the scattering estimate in the half-space $g_{e,4}^{\mathrm{hs}}$ was substantially different from zero. It is worth noting the adjacent TOR station where the bottom of the lower layer for *S* waves was at 70 km. We must warn that all conclusions derived from the 5-parameter inversion should be viewed as preliminary deductions.

DISCUSSION OF RESULTS

The inversion results can be regarded as the first direct evidence of great variations in lithospheric scattering with depth. The scattering contrast in the model of a layer of fixed thickness lying on a half-space was too high to derive a reliable scattering estimate in the half-space, i.e., in the depth interval of 35–140 km. Scattering estimates varied within 30% for the upper layer. The estimates of its thickness and scattering variation with depth are provisional. However, the fact that independent data for different stations and wave types are in good agreement lends support to our conclusions. It can be stated that the upper strongly scattering layer in Kamchatka is ~30–50 km thick, the upper 5–15 km usually showing higher scattering values. The exceptional case of lower scattering in the upper layer observed below the BER station can be explained by its position on a Paleozoic basement ridge, the situation unusual for Kamchatka.

It is clear from the above theory that our assumptions may cast some doubt on the accuracy of the absolute estimates. In particular, our numerical modeling [4] showed that the coefficient k in the relation $t_m^0 = g_e r^2/(kc)$ may differ slightly from the adopted value of 10.9 for models of varying scattering that are different from the model used. The related systematic errors can be roughly estimated as 10–20%. The qualitative conclusions and relative estimates obtained in this study can be taken as more reliable.

An important constraint on scattering models is a ratio between *P*- and *S*-waves scattering. The best data for deriving this ratio are the average values of *P*- and *S*-wave scattering in the 0–35 km layer. We found it to be 2.4 with a relative uncertainty within 10%.

The average scattering values themselves were 0.0192 km^{-1} (*P* waves) and 0.0082 km^{-1} (*S* waves) with a formal uncertainty within 5%; the inverse values yielded the effective free paths of 51 and 121 km, respectively.

CONCLUSIONS

1. An integral formula derived by Bocharov [3] was used to work out and test a method for estimating the vertical distribution of equivalent scattering in the lithosphere based on pulse broadening of *P* and *S* waves generated by near earthquakes.

2. Scattering was found to decline rapidly with depth. The upper strongly scattering layer was 30–50 km thick; its average scattering values were found to be ~0.02 km^{-1} for *P* wave and 0.008 km^{-1} for *S* waves. The scattering effect of this layer was too great to permit a reliable estimation of scattering below. The approximate estimate of the latter was an order of magnitude smaller.

3. The thickness of the scattering layer and the vertical variation of scattering in it were estimated for *P* and *S* waves beneath five stations. The scattering values vary from station to stations within 30%. There is some indication of vertical variation in the scattering layer, the scattering being higher in the upper 5–15 km.

4. The average scattering in the upper (0–35 km) lithosphere beneath Kamchatka was found, for frequencies of 2–6 Hz, to be 0.0195 km^{-1} for *P* waves and 0.0082 km^{-1} for *S* waves to within a formal uncertainty of about 5%. Considering that the theory is approximate, accidental experimental errors can make this value as high as 10–20%. The *P*- to *S*-wave scattering ratio was 2.4 to within 10%.

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REFERENCES

1. I. R. Abubakirov and A. A. Gusev, *Otsenka parametrov rasseyaniya obiemnykh voln v litosfere Kamchatki po ushireniyu impulsa s rasstoyaniem* (Estimation of body-wave scattering in the lithosphere of Kamchatka based on pulse broadening with distance) (Petropavlovsk–Kamchatskiy: IVGiG DVO RAN, Preprint 43, 1994).
2. A. A. Bocharov, *Izv. Vuzov. Radiofizika* **31**, N11: 1407–1409 (1988).
3. A. A. Bocharov, *Izv. Vuzov. Radiofizika* **33**, N4: 395–402 (1990).
4. A. A. Gusev and I. R. Abubakirov, *Ogibayushchie amplitud anizotropno rasseyannykh voln: modeli i realnost* (Amplitude envelopes of anisotropically scattered waves: models and realities)(Petropavlovsk–Kamchatskiy: IVGiG DVO RAN, Preprint 51, 1994).
5. A. A. Gusev and V. K. Lemzikov, *Volcanology and Seismology* (cover-to-cover translation of the Russian journal *Vulkanol. Seismol.*, 1983, N1: 94–108).
6. A. Ishimaru, *Wave Propagation and Scattering in Random Media. Vol. 1, Single Scattering and Transport Theory* (New York: Academic Press, 1978).
7. I. R. Abubakirov and A. A. Gusev, *Phys. Earth Planet. Inter.* **64**: 52–67 (1990).
8. K. Aki, *J. Geophys. Res.* **78**: 1334–1346 (1973).
9. J. Capon, *Bull. Seismol. Soc. Amer.* **64**: 235–266 (1974).
10. S. M. Flatte and R. S. Wu, *J. Geophys. Res.* **93**: 6601–6614 (1988).
11. L. Gao, Z. Hua, and R. Li, *Acta Seism. Sinica* **1**: 13–29 (1988).
12. M. Hoshiba, *J. Geophys. Res.* **98**: 15809–15824 (1993).
13. H. Sato, *J. Phys. Earth* **26**: 185–198 (1978).
14. H. Sato, *J. Geophys. Res.* **94**: 17735–17747 (1989).
15. I. P. Williamson, *Mon. Not. Roy. Astr. Soc.* **157**: 55–71 (1972).